CLASS 9 PHYSICS PREVIOUS YEAR QUESTIONS
FORCES & LAWS OF MOTION

Question-1

Find the remainder when $y^3 + y^2 - 2y + 5$ is divided by $y - 5$.

Solution:

$$
\begin{array}{r|l}
 & y^2 + 6y + 28 \\
\hline
y - 5 & y^3 + y^2 - 2y + 5 \\
 & -(y^3 - 5y^2) \\
\hline
 & \quad 6y^2 - 2y + 5 \\
 & \quad -(6y^2 - 30y) \\
\hline
 & \qquad 28y + 5 \\
 & \qquad -(28y - 140) \\
\hline
 & \qquad\qquad 145
\end{array}
$$

Remainder = 145

Again, we should evaluate p(5)

Let $p(y) = y^3 + y^2 - 2y + 5$

$\therefore p(5) = 5^3 + 5^2 - 2 \times 5 + 5$

$= 125 + 25 - 10 + 5$

$= 145$

Thus, we find that $p(5)$ is the remainder when

$p(y)$ is divided by $y - 5$.

Question-2

Determine the remainder when polynomial $p(x)$ is divided by $x - 2$.

Solution:

$p(x) = x^4 - 3x^2 + 2x - 5$

According to remainder theorem, the required remainder will be = $p(2)$

$p(x) = x^4 - 3x^2 + 2x - 5$

$\therefore p(2) = 2^4 - 3(2)^2 + 2(2) - 5$

$= 16 - 12 + 4 - 5 = 3$

Question-3

Show that $x + a$ is a factor of $x^n + a^n$ for any odd +ve integer n.

Solution:

Let $f(x) = x^n + a^n$

$x + a$ will be the factor of $x^n + a^n$ if $f(-a) = 0$

Now $f(-a) = (-a)^n + a^n = 0$ (since n is a odd +ve integer)

Thus $(x + a)$ is a factor of $x^n + a^n$.

When $f(x) = x^4 - 2x^3 + 3x^2 - ax$ is divided by $x + 1$ and $x - 1$, we get remainders as 19 and 5 respectively.
Find the remainder if $f(x)$ is divided by $x - 3$.

Solution:
When $f(x)$ is divided by $(x + 1)$ and $(x - 1)$, the remainders are 19 and 5 respectively.
$\therefore f(-1) = 19$ and $f(1) = 5$
$\Rightarrow (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b = 19$
$\Rightarrow 1 + 2 + 3 + a + b = 19$
$\therefore a + b = 13$ ----------- (i)

Again, $f(1) = 5$
$\Rightarrow 1^4 - 2 \times 1^3 + 3 \times 1^2 - a \times 1\ b = 5$
$\Rightarrow 1 - 2 + 3 - a + b = 5$
$\therefore b - a = 3$ ------------ (ii)

Solving eqn (i) and (ii), we get
$a = 5$ and $b = 8$
Now substituting the values of a and b in $f(x)$, we get
$\therefore f(x) = x^4 - 2x^3 + 3x^2 - 5x + 8$

Now $f(x)$ is divided by $(x - 3)$ so remainder will be $f(3)$
$\therefore f(x) = x^4 - 2x^3 + 3x^2 - 5x + 8$
$\Rightarrow f(3) = 3^4 - 2 \times 3^3 + 3 \times 3^2 - 5 \times 3 + 8$
$= 81 - 54 + 27 - 15 + 8 = 47$

What must be subtracted from $4x^4 - 2x^3 - 6x^2 + x - 5$ so that the result is exactly divisible by $2x^2 + 3x - 2$?

Solution:
Since $2x^2 + 3x - 2$ is of degree 2, so when

$p(x) = 4x^4 - 2x^3 - 6x^2 + x - 5$ is divided by

$q(x) = 2x^2 + 3x - 2$, the remainder should be a linear expression (degree of remainder < degree of divisor).

Let the remainder $r(x) = ax + b$ for exact division this remainder should be subtracted from $p(x)$ Now let

$f(x) = p(x) - r(x)$
$= 4x^4 - 2x^3 - 6x^2 + x - 5 - (ax + b)$
$f(x) = 4x^4 - 2x^3 - 6x^2 + (1 - a)x - 5 - b$

Again, we have
$q(x) = 2x^2 + 3x - 2$
$= 2x^2 + 4x - x - 2$
$= 2x(x + 2) - 1(x + 2)$
$= (x + 2)(2x - 1)$

Since $x + 2$ and $2x - 1$ are factors of $q(x)$ and $f(x)$ is exactly divisible by q(x), hence (x + 2) and (2x - 1) are also factors of f(x), i.e.

$f(-2) = 0$ and $f(\frac{1}{2}) = 0$

$\therefore f(-2)=4(-2)^4 - 2(-2)^3 - 6(-2)^2 + (1 - a)(-2) - b - 5 = 0$

$\Rightarrow 64 + 16 - 24 - 2 + 2a - b - 5 = 0$

$\Rightarrow 2a - b = -49$ ...... (i)

Again $f(\frac{1}{2}) = 0$

$\Rightarrow 4(\frac{1}{2})^4 - 2(\frac{1}{2})^3 - 6(\frac{1}{2})^2 + (1 - a) \times \frac{1}{2} - b - 5 = 0$

$\Rightarrow -\frac{1}{4} - \frac{3}{2} + \frac{1}{2} - \frac{a}{2} - b - 5 = 0$

$\Rightarrow -\frac{a}{2} - b - 6 = 0$

$\Rightarrow a + 2b = -12$ ...... (ii)

Solving eqns (i) and (ii), we get $a = -22$ and $b = 5$.

$\therefore r(x) = -22x + 5$ should be subtracted.

Question-6

If both $(x + 1)$ and $(x - 1)$ are factors of $ax^3 + x^2 - 2x + b$, find a and b.

Solution:

Let $p(x) = ax^3 + x^2 - 2x + b$

Since $(x + 1)$ and $(x - 1)$ are the factors of $p(x)$,

$\therefore p(-1) = 0$ and $p(1) = 0$

$\therefore p(-1) = a(-1)^3 + (-1)^2 - 2(-1) + b = 0$

$\Rightarrow -a + 1 + 2 + b = 0$

$\Rightarrow a - b = 3$ ...... (i)

Again, $p(1) = a(1)^3 + (1)^2 - 2(1) + b = 0$

$\Rightarrow a + 1 - 2 + b = 0$

$\Rightarrow a + b = 1$ ...... (ii)

Solving equations (i) and (ii) we get $a = 2$ and $b = -1$

Question-7

If x^2 - 1 is a factor of $ax^4 + bx^3 + cx^2 + dx + e$, show that $a + c + e = b + d = 0$.

Solution:

Since $x^2 - 1 = (x + 1)(x - 1)$ is a factor of

$p(x) = ax^4 + bx^3 + cx^2 + dx + e$

$\therefore$ $p(x)$ is divisible by $(x + 1)$ and $(x - 1)$ separately

$\Rightarrow p(1) = 0$ and $p(-1) = 0$

$p(1) = a(1)^4 + b(1)^3 + c(1)^2 + d(1) + e = 0$

$\Rightarrow a + b + c + d + e = 0$ ......(i)

Similarly, $p(-1) = a(-1)^4 + b(-1)^3 + c(-1)^2 + d(-1) + e = 0$

$\Rightarrow a - b + c - d + e = 0$

$\Rightarrow a + c + e = b + d$ ......(ii)

Putting the value of $a + c + e$ in eqn (i), we get

$a + b + c + d + e = 0$

$\Rightarrow a + c + e + b + d = 0$

$\Rightarrow b + d + b + d = 0$

$\Rightarrow 2(b + d) = 0$

$\Rightarrow b + d = 0$ ......(iii)

Comparing equations (ii) and (iii), we get

$a + c + e = b + d = 0$

Question-8

Factorise each of the following expressions:

(i) $48x^3 - 36x^2$

(ii) $5x^2 - 15xy$

(iii) $15x^3y^2z - 25xy^2z^3$

Solution:

(i) $48x^3 - 36x^2$

$= 12.4.x^2.x - 12.3.x^2$

$= 12x^2(4x - 3)$

(ii) $5x^2 - 15xy = 5x(x - 3y)$

(iii) $15x^3y^2z - 25xy^2z^3 = 5xy^2z\,(3x^2 - 5z^2)$

Question-9

Factorise

(i) $2x^2(x + y) - 3(x + y)$

(ii) $5xy(5x + y) - 5y(5x + y)$

(iii) $x(x^2 + y^2 - z^2) + y(x^2 + y^2 - z^2) + z(x^2 + y^2 - z^2)$

(iv) $ab(a^2 + b^2 - c^2) + bc(a^2 + b^2 - c^2) + ca(a^2+b^2-c^2)$

Solution:

(i) $2x^2(x + y) - 3(x + y)$

$= (x + y)(2x^2 - 3)$

(ii) $5xy(5x + y) - 5y(5x + y)$

$= 5y(5x + y)(x - 1)$

(iii) $x(x^2 + y^2 - z^2) + y(x^2 + y^2 - z^2) + z(x^2 + y^2 - z^2)$

$= (x^2 + y^2 - z^2)(x + y + z)$

(iv) $ab(a^2 + b^2 - c^2) + bc(a^2+b^2-c^2) + ca(a^2+b^2 - c^2)$

$= (a^2 + b^2 - c^2)(ab + bc + ca)$

Factorise

(i) $6ab - b^2 + 12ac - 2bc$

(ii) $x^2 + y - xy - x$

(iii) $x^3 - 2x^2y + 3xy^2 - 6y^3$

(iv) $a(a + b - c) – bc$

Solution:

(i) $6ab - b^2 + 12ac - 2bc$

$= (6ab - b^2) + (12ac - 2bc)$

$= b(6a - b) + 2c(6a - b)$

$= (6a - b)(b + 2c)$

(ii) $x^2 + y - xy - x$

$= x^2 - xy - x + y$

$= x(x - y) - (x - y)$

$= (x - y)(x - 1)$

(iii) $x^3 - 2x^2y + 3xy^2 - 6y^3$

$= (x^3 - 2x^2y) + (3xy^2 - 6y^3)$

$= x^2(x - 2y) + 3y^2(x - 2y)$

$= (x - 2y)(x^2 + 3y^2)$

(iv) $a(a + b - c) - bc$

$= a^2 + ab - ac - bc$

$= a(a + b) - c(a + b)$

$= (a + b)(a - c)$

Question-11

Factorise each of the following expressions.

(i) $25x^2y^2 - 20xy^2z + 4y^2z^2$

(ii) $4x^2 - 4\sqrt{7}x + 7$

(iii) $\frac{a^2}{b^2} + 2 + \frac{b^2}{a^2}$

(iv) $4a^2 + 12ab + 9b^2 - 8a - 12b$

Solution:

(i) $25x^2y^2 - 20xy^2z + 4y^2z^2$

$= (5xy)^2 - 2\times5xy \times 2yz + (2yz)^2$

$= (5xy + 2yz)^2$

$= y^2 (5x + 2z)^2$

(ii) $4x^2 - 4\sqrt{7}x + 7$

$= (2x)^2 + 2\times2x \times \sqrt{7} + (\sqrt{7})^2 = (2x + \sqrt{7})^2$

(iii) $\frac{a^2}{b^2} + 2 + \frac{b^2}{a^2}$

$= (\frac{a}{b})^2 + 2 \times \frac{a}{b} \times \frac{b}{a} + (\frac{b}{a})^2 = (\frac{a}{b} + \frac{b}{a})^2$

(iv) $4a^2 + 12ab + 9b^2 - 8a - 12b$

$= (4a^2 + 12ab + 9b^2) - 4(2a + 3b)$

$= (2a)^2 + 2 \times 2a \times 3b + (3b)^2 - 4(2a + 3b)$

$= (2a + 3b)^2 - 4(2a + 3b)$

$= (2a + 3b)(2a + 3b - 4)$

Question-12

Factorise

(i) $25x^2 - 36y^2$

(ii) $2ab - a^2 - b^2 + 1$

(iii) $36a^2 - 12a + 1 - 25b^2$

(iv) $a^4 - 81b^4$

(v) $a^{12}b^4 - a^4b^{12}$

(vi) $4x^2 - 9y^2 - 2x - 3y$

Solution:

(i) $25x^2 - 36y^2$

$= (5x)^2 - (6y)^2$

$= (5x + 6y)(5x - 6y)$

(ii) $2ab - a^2 - b^2 + 1$

$= 1 - (a^2 + b^2 - 2ab)$

$= 1 - (a - b)^2$

$= (1 + a - b)(1 - a + b)$

(iii) $36a^2 - 12a + 1 - 25b^2$

$= (6a)^2 - 2 \times 6a \times 1 + 1^2 - (5b)^2$

$= (6a - 1)^2 - (5b)^2$

$= (6a - 1 + 5b)(6a - 1 - 5b)$

$= (6a + 5b - 1)(6a - 5b - 1)$

(iv) $a^4 - 81b^4$

$= (a^2)^2 - (9b^2)^2$

$= (a^2 + 9b^2)(a^2 - 9b^2)$

$= (a^2 + 9b^2)(a + 3b)(a - 3b)$

(v) $a^{12} b^4 - a^4 b^{12}$

$= a^4b^4 (a^8 - b^8)$

$= a^4b^4 [(a^4)^2 - (b^4)^2]$

$= a^4b^4 (a^4 + b^4)(a^4 - b^4)$

$= a^4b^4 (a^4 + b^4)(a^2 + b^2)(a^2 - b^2)$

$= a^4b^4 (a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$

(vi) $4x^2 - 9y^2 - 2x - 3y$

$= (2x)^2 - (3y)^2 - (2x + 3y)$

$= (2x + 3y)(2x - 3y) - (2x + 3y)$

$= (2x + 3y)(2x - 3y - 1)$

Question-13

Factorise by completing the squares.

(i) $a^4 + a^2 + 1$

(ii) $y^4 + 5y^2 + 9$

(iii) $x^4 + 4$

(iv) $x^4 + 4x^2 + 3$

Solution:

(i) $a^4 + a^2 + 1$

$= a^4 + a^2 + 1 + a^2 - a^2$

$= a^4 + 2a^2 + 1 - a^2$

$= (a^2)^2 + 2a^2 + 1 - a^2$

$= (a^2 + 1)^2 - a^2$

$= (a^2 + 1 + a)\ (a^2 + 1 - a)$

$= (a^2 + a + 1)\ (a^2 - a + 1)$

(ii) $y^4 + 5y^2 + 9$

$= (y^2)^2 + 5y^2 + (3)^2 + y^2 - y^2$

$= (y^2)^2 + 6y^2 + (3)^2 - y^2$

$= (y^2 + 3)^2 - y^2$

$= (y^2 + y + 3)\ (y^2 - y + 3)$

(iii) $x^4 + 4$

$= x^4 + 4 + 4x^2 - 4x^2$

$= (x^2)^2 + (2)^2 + (2x)^2 - 4x^2$

$= (x^2 + 2)^2 - (2x)^2$

$= (x^2 + 2x + 2)\ (x^2 - 2x + 2)$

(iv) $x^4 + 4x^2 + 3$

$= x^4 + 4x^2 + 4 - 1$

$= x^4 + 4x^2 + 3 + 1 - 1$

$= (x^2)^2 + 2 \times 2x^2 + (2)^2 - 1$

$= (x^2 + 2)^2 - 1$

$= (x^2 + 2 + 1)\ (x^2 + 2 - 1) = (x^2 + 3)\ (x^2 + 1)$

Factorise the following expressions:

(i) $a^3 - 27$

(ii) $1 - 27x^3$

(iii) $8x^3 - (2x - 3y)^3$

(iv) $a^8 - a^2b^6$

(v) $a^3 - 5\sqrt{5}b^3$

Solution:

(i) $a^3 - 27$

$= a^3 - (3)^3$

$= (a - 3)(a^2 + 3a + 9)$

(ii) $1 - 27x^3 = 1^3 - (3x)^3$

$= (1 - 3x)[1^2 + 1 \times 3x + (3x)^2]$

$= (1 - 3x)(1 + 3x + 9x^2)$

(iii) $8x^3 - (2x - 3y)^3 = (2x)^3 - (2x - 3y)^3$

$= [2x - (2x - 3y)][(2x)^2 + 2x(2x - 3y) + (2x - 3y)^2]$

$= (2x-2x+ 3y)(4x^2 + 4x^2- 6xy + 4x^2+ 9y^2-12xy)$

$= 3y(12x^2 + 9y^2 - 18xy)$

$= 3y\times3(4x^2 + 3y^2 - 6xy)$

$= 9y(4x^2 + 3y^2 - 6xy)$

(iv) $a^8 - a^2b^6 = a^2(a^6 - b^6)$

$= a^2[(a^3)^2 - (b^3)^2]$

$= a^2(a^3 + b^3)(a^3 - b^3)$

$= a^2(a + b)(a^2 - ab + b^2)(a - b)(a^2 + ab + b^2)$

$= a^2(a + b)(a - b)(a^2 - ab + b^2)(a^2 + ab + b^2)$

(v) $a^3 - 5\sqrt{5}b^3 = a^3 - (\sqrt{5}b)^3$

$= (a- \sqrt{5}b)(a^2+\sqrt{5}ab+5b^2)$

Factorise the following

(i) $16p^3q^3 + 54r^3$

(ii) $\frac{a^3}{8} + 8b^3$

(iii) $2\sqrt{2}a^3 + 3\sqrt{3}b^3$

(iv) $8a^4b + \frac{1}{125}ab^4$

(v) $a^7 - 64a$

Solution:

(i) $16p^3q^3 + 54r^3$

$= 2(8p^3q^3 + 27r^3)$

$= 2[(2pq)^3 + (3r)^3]$

$= 2(2pq + 3r)\,[(2pq)^2 - 2pq\,.\,3r + (3r)^2]$

$= 2\,(2pq + 3r)\,(4p^2q^2 - 6pqr + 9r^2)$

(ii) $\frac{a^3}{8} + 8b^3 = (\frac{a}{2})^3 + (2b)^3$

$= (\frac{a}{2} + 2b)(\frac{a^2}{4} - ab + 4b^2)$

(iii) $2\sqrt{2}a^3 + 3\sqrt{3}b^3$

$= (\sqrt{2}a)^3 + (\sqrt{3}b)^3$

$= (\sqrt{2}a + \sqrt{3}b)(2a^2 - \sqrt{6}ab + 3b^2)$

(iv) $8a^4b + \frac{1}{125}ab^4 = ab(8a^3 + \frac{b^3}{125})$

$= ab[(2a)^3 + \left(\frac{b}{5}\right)^3]$

$= ab\,(2a + \frac{b}{5})(4a^2 - \frac{2}{5}ab + \frac{b^2}{25})$

(v) $a^7 - 64a$

$= a(a^6 - 64)$

$= a(a^6 - 2^6)$

$= a[(a^3)^2 - (2^3)^2]$

$= a(a^3 + 2^3)\,(a^3 - 2^3)$

$= a(a + 2)\,(a^2 - 2a + 4)\,(a - 2)\,(a^2 + 2a + 4)$

$= a(a + 2)\,(a - 2)\,(a^2 - 2a + 4)\,(a^2 + 2a + 4)$

Question-16

Factorise

(i) $x^3 + 9x^2 + 27x + 27$

(ii) $x^3 - 9x^2y + 27xy^2 - 27y^3$

Solution:

(i) $x^3 + 9x^2 + 27x + 27$

$= (x)^3 + 3(x)^2(3) + 3(x)(3)^2 + (3)^3$

$= (2x + 3)^3$

(ii) $x^3 - 9x^2y + 27xy^2 - 27y^3$

$= x^3 - 3(x)^2(3y) + 3(x)(3y)^2 - (3y)^3$

$= (x - 3y)^3$

Using identities, find the value of
(i) 101^2
(ii) 98^2
(iii) $(0.98)^2$
(iv) 11×9
(v) $190 \times 190 - 10 \times 10$

Solution:

(i) $101^2 = (100 + 1)^2$
$= 100^2 + 2 \times 100 \times 1 + 1^2$
$= 10000 + 200 + 1 = 10201$

(ii) $98^2 = (100 - 2)^2$
$= 100^2 - 2 \times 100 \times 2 + 2^2$
$= 10000 - 400 + 4 = 9604$

(iii) $0.98^2 = (1 - 0.02)^2$
$= 1^2 - 2 \times 1 \times 0.02 + (0.02)^2$
$= 1 - 0.04 + 0.0004$
$= 0.9604$

(iv) $11 \times 9 = (10+1)(10-1)$
$= 10^2 - 1^2$
$= 99$

(v) $190 \times 190 - 10 \times 10 = 190^2 - 10^2$
$= (190 + 10)(190 - 10)$
$= 200 \times 180$
$= 36000$

Question-18

If $x + y = 5$ and $xy = 4$, find $x - y$, using identities.

Solution:

$(x - y)^2 = x^2 - 2xy + y^2$
$= x^2 + 2xy + y^2 - 4xy$
$= (x + y)^2 - 4xy$
$= 5^2 - 4 \times 4$
$= 25 - 16 = 9$
$\therefore\ x - y = \sqrt{9}$
$= 3$

Expand

(i) $(x + 5y + 6z)^2$

(ii) $(2a - 3b + 4c)^2$

(iii) $(-a + 6b + 5c)^2$

(iv) $(-p + 4q - 3r)^2$

Solution:

Using the identity

$(x + y + z)^2 = x^2 + y^2 + 2xy + 2yz + 2xz$, we have

(i) $(x + 5y + 6z)^2$

$= (x)^2 + (5y)^2 + (6z)^2 + 2(x)(5y) + 2(5y)(6z) + 2(x)(6z)$

$= x^2 + 25y^2 + 36z^2 + 10xy + 60yz + 12\,xz$

(ii) $(2a - 3b + 4c)^2$

$= (2a)^2+(-3b)^2+(4c)^2+2(2a)(-3b)+2(-3b)(4c)+2(2a)(4c)$

$= 4a^2+ 9b^2 + 16c^2 - 12ab - 24bc + 16ac$

(iii) $(-a + 6b + 5c)^2$

$= (-a)^2+(6b)^2+(5c)^2+2(-a)(6b)+2(6b)(5c)+2(-a)(5c)$

$= a^2 + 36b^2 + 25c^2 - 12ab + 60bc - 10ac$

(iv) $(-p + 4q - 3r)^2$

$= (-p)^2+(4q)^2+(-3r)^2+2(-p)(4q)+2(4q)(-3r)+2(-p)(-3r)$

$= p^2 + 16q^2 + 9r^2 - 8pq - 24qr + 6pr$

Question-20

Expand

(i) $(2x + 5y)^3$

(ii) $(5p - 3q)^3$

(iii) $(-a + 2b)^3$

Solution:

(i) Using the identity

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ we get

$(2x + 5y)^3 = (2x)^3 + 3(2x)^2\,(5y) + 3(2x)(5y)^2 + (5y)^3$

$= 8x^3 + 60x^2y + 150xy^2 + 25y^3$

(ii) Using the identity

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$ we get

$(5p - 3q)^3 = (5p)^3-3(5p)^2(3q)+3(5p)(3q)^2-(3q)^3$

$= 125p^3 - 225p^2q + 135pq^2 - 27q^3$

(iii) Using the identity for $(x + y)^3$ we get

$(-a + 2b)^3 = (-a)^3+3(-a)^2(2b)+3(-a)(2b)^2+(2b)^3$

$= a^3 + 6a^2b - 12ab^2 + 8b^3$

Question-21

Evaluate, using identities
(i) 102^3
(ii) 99^3

Solution:

(i) $102^3 = (100 + 2)^3$
$= (100)^3 + 3(100)^2 (2) + 3(100)(2^2) + 2^3$
$= 10,00,000 + 60,000 + 1200 + 8$
$= 10,61,208$

(ii) $99^3 = (100 - 1)^3$
$= 100^3 - 3(100)^2(1) + 3(100)(1^2) - 1^3$
$= 10,00,000 - 30,000 + 300 - 1$
$= 9,70,299$

Question-22

Simplify $(2a + b)^3 + (2a - b)^3$

Solution:

$(2a + b)^3 + (2a - b)^3 = (8a^3 + 12a^2b + 3ab^2 + b^3) + (8a^3 - 12a^2b + 3ab^2 - b^3)$
$= 16a^3 + 6ab^2$

Question-23

Evaluate, using the identities
(i) 505 x 503,
(ii) 37 x 26

Solution:

(i) $505 \times 503 = (500 + 5)(500 + 3)$
$= 500^2 + (5 + 3)(500) + 5 \times 3$
$= 250000 + 4000 + 15 = 254015$

(ii) $37 \times 26 = (30 + 7)(30 - 4)$
$= 30^2 + (7 - 4)(30) - 7 \times 4$
$= 900 + 90 - 28 = 962$

Question-24

Find the products
(i) $(2a + 5b)(4a^2 - 10ab + 25b^2)$
(ii) $(2a - 5b)(4a^2 + 10ab + 25b^2)$

Solution:

(i) $(2a + 5b)(4a^2 - 10ab + 25b^2)$
$= (2a + 5b)[(2a)^2 - (2a)(5b) + (5b)^2]$
$= (2a)^3 + (5b)^3$ (from identity 10 above)
$= 8a^3 + 125b^3$

(ii) $(2a - 5b)(4a^2 + 10ab + 25b^2)$
$= (2a - 5b)[(2a)^2 + (2a)(5b) + (5b)^2]$
$= (2a)^3 - (5b)^3$ (from identity 11 above)
$= 8a^3 - 125b^3$

Question-25

Factorise
(i) $30x^3y + 24x^2y^2 - 6xy$
(ii) $5x(a - b) + 6y(a - b)$

Solution:

(i) The greatest monomial that is a common factor of the three terms is *6xy*.
$\therefore \quad 30x^3y + 24x^2y^2 - 6xy = 6xy(5x^2 + 4xy - 1)$

(ii) Here the polynomial $(a - b)$ is a common factor.
$\therefore 5x(a - b) + 6y(a - b) = (a - b)(5x + 6y)$

Question-26

Factorise
(i) $7a^3 + 7a - 2a^2 - 2$
(ii) $4ax + 3by - 3ay - 4bx$
(iii) $x^3 + y^3 + x^2y + xy^2$

Solution:

(i) $7a^3 + 7a - 2a^2 - 2 = 7a(a^2 + 1) - 2(a^2 + 1)$
$= (a^2 + 1)(7a - 2)$

(ii) The terms of $4ax + 3by - 3ay - 4bx$ can be rearranged and factorized.
$4ax - 4bx - 3ay + 3by = 4x(a - b) - 3y(a - b)$
$= (a - b)(4x - 3y)$

(iii) $x^3 + y^3 + x^2y + xy^2 = x^3 + x^2y + xy^2 + y^3$
$= x^2(x + y) + y^2(x + y)$
$= (x + y)(x^2 + y^2)$

Question-27

Factorise

(i) $\boldsymbol{9x^2 + 30xy + 25y^2}$

(ii) $\boldsymbol{9x^2 - 30xy + 25y^2}$

Solution:

(i) $9x^2 + 30xy + 25y^2$

$= (3x)^2 + 2(3x)(5y) + (5y)^2$

$= (3x + 5y)^2$

(ii) $9x^2 - 30xy + 25y^2$

$= (3x)^2 - 2(3x)(5y) + (5y)^2$

$= (3x - 5y)^2$

Question-28

Factorize

(i) $\boldsymbol{9x^2 - y^2}$

(ii) $\boldsymbol{(3 - x)^2 - 36x^2}$

(iii) $\boldsymbol{(2x - 3y)^2 - (3y + 4y)^2}$

(iv) $\boldsymbol{16x^4 - y^4}$

Solution:

(i) $9x^2 - y^2 = (3x)^2 - (y)^2$

$= (3x + y)(3x - y)$

(ii) $(3 - x)^2 - 36x^2 = (3 - x)^2 - (6x)^2$

$= (3 - x + 6x)(3 - x - 6x)$

$= (3 + 5x)(3 - 7x)$

(iii) $(2x - 3y)^2 - (3x + 4y)^2 = (2x - 3y + 3x + 4y)(2x - 3y - 3x - 4y)$

$= (5x + y)(- x - 7y)$

$= -(5x + y)(x + 7y)$

(iv) $16x^4 - y^4 = (4x^2)^2 - (y^2)^2$

$= (4x^2 + y^2)(4x^2 - y^2)$

$= (4x^2 + y^2)(2x + y)(2x - y)$

Question-29

Factorise
(i) $\boldsymbol{x^2 + y^2 + 9z^2 + 2xy + 6yz + 6xz,}$
(ii) $\boldsymbol{x^2 + 4y^2 + 9z^2 - 4xy - 12yz + 6xz}$

Solution:

(i) $x^2 + y^2 + 9z^2 + 2xy + 6yz + 6xz$
$= x^2+(y)^2+(3z)^2+ 2(x)(y)+2(y)(3z)+2(x)(3z)$
$= (x + y + 3z)^2$

(ii) $x^2 + 4y^2 + 9z^2 - 4xy - 12yz + 6xz$
$= x^2 + (2y)^2 + (3z)^2 - 2(x)(2y) - 2(2y)(3z)+ 2(x)(3z)$
$= x^2 + (2y)^2 + (3z)^2 + 2(x)(-2y) + 2(-2y)(3z) + 2(x)(3z)$
$= (x - 2y + 3z)^2$

Question-30

Factorise $\boldsymbol{x^6 - y^6}$

Solution:

$x^6 - y^6 = (x^2)^3 - (y^2)^3$
$= (x^2 - y^2)[(x^2)^2 + x^2y^2 +(y^2)^2]$
$= (x + y)(x - y)(x^4 + x^2y^2 + y^4)$

(Here the third factor can be further factorised as shown)
$= (x + y)(x - y)(x^4 + 2x^2y^2 + y^4 - x^2y^2)$
$= (x + y)(x - y)[(x^2 + y^2)^2 - x^2y^2]$
$= (x + y)(x - y)\ (x^2 + y^2 + xy)(x^2 + y^2 - xy)$