CLASS 9 MATHS PREVIOUS YEAR QUESTIONS
TRIANGLES

Very Short Answer Type

Question 1. Find the measure of each exterior angle of an equilateral triangle.
Solution: We know that each interior angle of an equilateral triangle is 60°.
∴ Each exterior angle = 180° – 60° = 120°

Question 2. If in ΔABC, ∠A = ∠B + ∠C, then write the shape of the given triangle.
Solution: Here, ∠A = ∠B + ∠C
And in ΔABC, by angle sum property, we have
∠A + ∠B + C = 180°
⇒ ∠A + ∠A = 180°
⇒ 2∠A = 180°
⇒ ∠A = 90°
Hence, the given triangle is a right triangle.

Question 3. In ΔPQR, PQ = QR and ∠R = 50°, then find the measure of ∠Q.
Solution: Here, in ΔPQR, PQ = QR
⇒ ∠R = ∠P = 50° (given)
Now, ∠P + ∠Q + ∠R = 180°
50° + ∠Q + 50° = 180°
⇒ ∠Q = 180° – 50° – 50°
= 80°

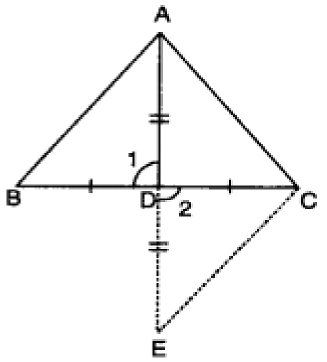
Question 4. If ΔSKY ≅ ΔMON by SSS congruence rule, then write three equalities of corresponding angles.
Solution: Since ΔSKY ≅ ΔMON by SSS congruence rule, then three equalities of corresponding angles are ∠S = ∠M, ∠K = ∠O and ∠Y = ∠N.

Question 5. Is ΔABC possible, if AB = 6 cm, BC = 4 cm and AC = 1.5 cm ?
Solution: Since 4 + 1.5 = 5.5 ≠ 6
Thus, triangle is not possible.

Question 6. In ΔMNO, if ∠N = 90°, then write the longest side.
Solution: We know that, side opposite to the largest angle is longest.
∴ Longest side = MO.

Question 7. In ΔABC, if AB = AC and ∠B = 70°, find ∠A.
Solution: Here, in ΔABC AB = AC ∠C = ∠B [∠s opp. to equal sides of a Δ)
Now, ∠A + ∠B + ∠C = 180°
⇒ ∠A + 70° + 70° = 180° [∵ ∠B = 70°]
⇒ ∠A = 180° – 70° – 70° = 40°

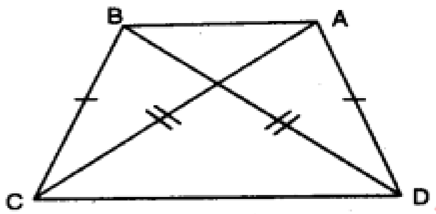
Question 8. In ΔABC, if AD is a median, then show that AB + AC > 2AD.



Solution:
Produce AD to E, such that AD = DE.
In ΔADB and ΔEDC, we have
BD = CD, AD = DE and ∠1 = ∠2
ΔADB ≅ ΔEDC
AB = CE
Now, in ΔAEC, we have
AC + CE > AE
AC + AB > AD + DE
AB + AC > 2AD [∵ AD = DE]

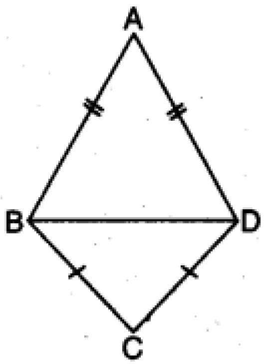
Short Answer Type I

Question 1. In the given figure, AD = BC and BD = AC, prove that ∠DAB = ∠CBA.
Solution:



In ΔDAB and ΔCBA, we have
AD = BC [given]
BD = AC [given]
AB = AB [common]
∴ ΔDAB ≅ ΔCBA [by SSS congruence axiom]
Thus, ∠DAB =∠CBA [c.p.c.t.]

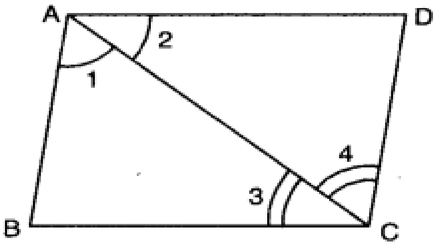
Question 2. In the given figure, ΔABD and ABCD are isosceles triangles on the same base BD. Prove that ∠ABC = ∠ADC.
Solution:



In ΔABD, we have
AB = AD (given)
∠ABD = ∠ADB [angles opposite to equal sides are equal] …(i)

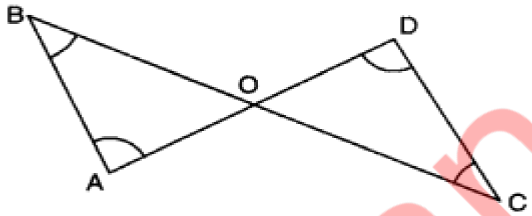
In ΔBCD, we have
$CB = CD$
$\Rightarrow \angle CBD = \angle CDB$ [angles opposite to equal sides are equal] … (ii)
Adding (i) and (ii), we have
$\angle ABD + \angle CBD = \angle ADB + \angle CDB$
$\Rightarrow \angle ABC = \angle ADC$

Question 3. In the given figure, if $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$, then prove that BC = CD.
Solution:



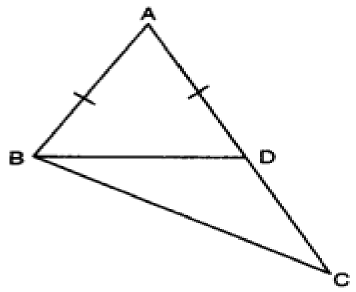
In ΔABC and ACDA, we have
$\angle 1 = \angle 2$ (given)
AC = AC [common]
$\angle 3 = \angle 4$ [given]
So, by using ASA congruence axiom
$\Delta ABC \cong \Delta CDA$
Since corresponding parts of congruent triangles are equal
$\therefore$ BC = CD

Question 4. In the given figure, $\angle B < \angle A$ and $\angle C < \angle D$. Show that AD < BC.



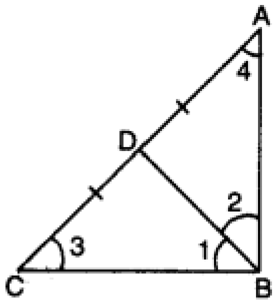
Solution:
Here, $\angle B < \angle A$
$\Rightarrow$ AO < BO …..(i)
and $\angle C < \angle D$
$\Rightarrow$ OD < CO …..(ii)
[$\therefore$ side opposite to greater angle is longer]
Adding (i) and (ii), we obtain
AO + OD < BO + CO
AD < BC

Question 5. In the given figure, AC > AB and D is a point on AC such that AB = AD. Show that BC > CD.
Solution:



Here, in ΔABD, AB = AD
∠ABD = ∠ADB [∠s opp. to equal sides of a Δ]
In ΔBAD
ext. ∠BDC = ∠BAD + ∠ABD
⇒ ∠BDC > ∠ABD ….(ii)
Also, in ΔBDC .
ext. ∠ADB > ∠CBD …(iii)
From (ii) and (iii), we have
∠BDC > CD [∵ sides opp. to greater angle is larger]

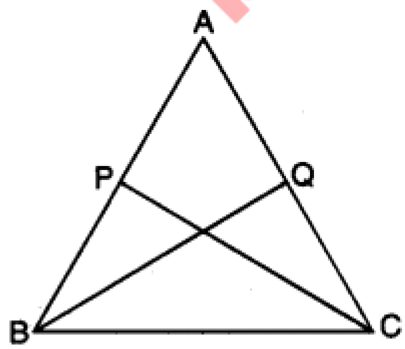
Question 6. In a triangle ABC, D is the mid-point of side AC such that BD = 12 AC. Show that ∠ABC is a right angle.
Solution:



Here, in ΔABC, D is the mid-point of AC.
⇒ AD = CD = 12AC …(i)
Also, BD = 12AC… (ii) [given]
From (i) and (ii), we obtain
AD = BD and CD = BD
⇒ ∠2 = ∠4 and ∠1 = ∠3 …..(iii)
In ΔABC, we have
∠ABC + ∠ACB + ∠CAB = 180°
⇒ ∠1 + ∠2 + ∠3 + ∠4 = 180°
⇒ ∠1 + ∠2 + ∠1 + ∠2 = 180° [using (iii)]
⇒ 2(∠1 + ∠2) = 180°
⇒ ∠1 + ∠2 = 90°
Hence, ∠ABC = 90°

Short Answer Type II

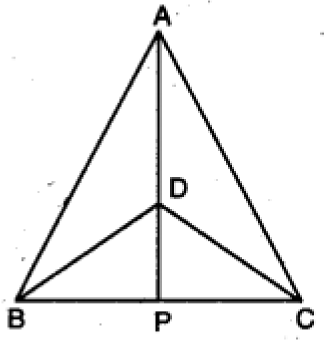
Question 1. ABC is an isosceles triangle with AB = AC. P and Q are points on AB and AC respectively such that AP = AQ. Prove that CP = BQ.
Solution:



In ΔABQ and ΔACP, we have
AB = AC (given)
∠BAQ = ∠CAP [common]
AQ = AP (given)

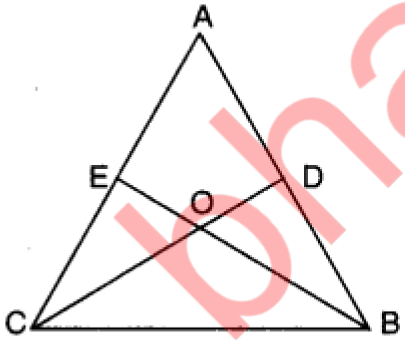
∴ By SAS congruence criteria, we have
ΔABQ ≅ ΔACP
CP = BQ

Question 2. In the given figure, ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC, AD is extended to intersect BC at P. Show that : (i) ΔABD ≅ ΔACD (ii) ΔABP ≅ ΔACP



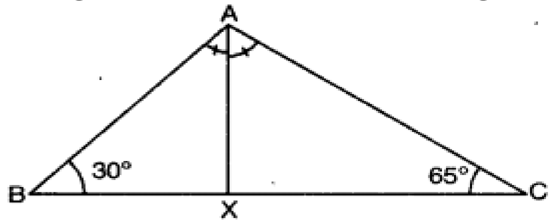
Solution:
(i) In ΔABD and ΔACD
AB = AC [given]
BD = CD [given]
AD = AD [common)]
∴ By SSS congruence axiom, we have
ΔABD ≅ ΔACD
(ii) In ΔABP and ΔACP
AB = AC [given]
∠BAP = ∠CAP [c.p.cit. as ΔABD ≅ ΔACD]
AP = AP [common]
∴ By SAS congruence axiom, we have
ΔABP ≅ ΔACP

Question 3. In the given figure, it is given that AE = AD and BD = CE. Prove that ΔAEB ≅ ΔADC.



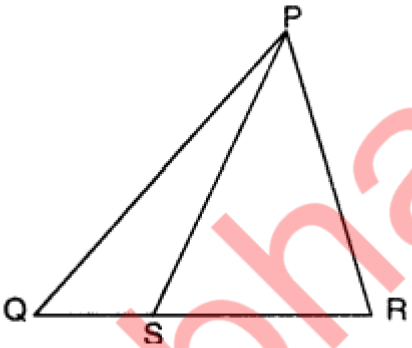
Solution:
We have AE = AD … (i)
and CE = BD … (ii)
On adding (i) and (ii),
we have AE + CE = AD + BD
⇒ AC = AB
Now, in ΔAEB and ΔADC,
we have AE = AD [given]
AB = AC [proved above]
∠A = ∠A [common]
∴ By SAS congruence axiom, we have
ΔAEB = ΔADC

Question 4. In the given figure, in ∆ABC, ∠B = 30°, ∠C = 65° and the bisector of ∠A meets BC in X. Arrange AX, BX and CX in ascending order of magnitude.



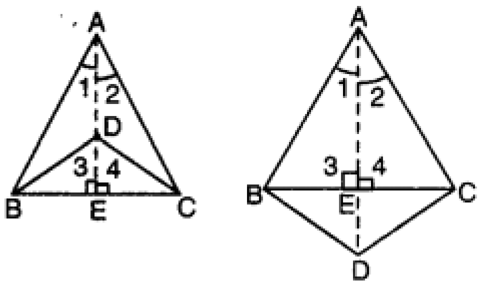
Solution:
Here, AX bisects ∠BAC.
∴ ∠BAX = ∠CAX = x (say)
Now, ∠A + ∠B + C = 180° [angle sum property of a triangle]
⇒ 2x + 30° + 65° = 180°
⇒ 2x + 95 = 180°
⇒ 2x = 180° – 95°
⇒ 2x = 85°
⇒ x = 85∘2 = 42.59
In ∆ABX, we have x > 30°
BAX > ∠ABX
⇒ BX > AX (side opp. to larger angle is greater)
⇒ AX < BX
Also, in ∆ACX, we have 65° > x
⇒ ∠ACX > ∠CAX
⇒ AX > CX [side opp. to larger angle is greater]
⇒ CX > AX … (ii)
Hence, from (i) and (ii), we have
CX < AX < BX

Question 5. In figure, 'S' is any point on the side QR of APQR. Prove that PQ + QR + RP > 2PS.



Solution:
In ∆PQS, we have
PQ + QS > PS …(i)
[∵ sum of any two sides of a triangle is greater than the third side]
In ∆PRS, we have
RP + RS > PS …(ii)
Adding (i) and (ii), we have
PQ + (QS + RS) + RP > 2PS
Hence, PQ + QR + RP > 2PS. [∵ QS + RS = QR]

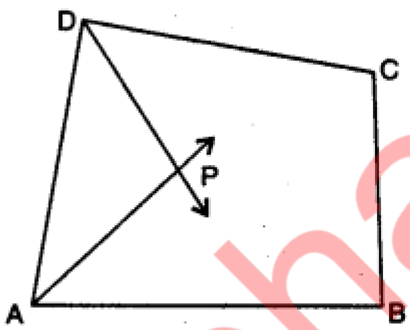
Question 6. If two isosceles triangles have a common base, prove that the line joining their vertices bisects them at right angles.
Solution: Here, two triangles ABC and BDC having the common
base BC, such that AB = AC and DB = DC.
Now, in ∆ABD and ∆ACD



AB = AC [given]
BD = CD [given]
AD = AD [common]
∴ ΔABD ≅ ΔACD [by SSS congruence axiom]
⇒ ∠1 = ∠2 [c.p.c.t.]
Again, in ΔABE and ΔACE, we have
AB = AC [given]
∠1 = ∠2 [proved above]
AE = AE [common]
ΔABE = ΔACE [by SAS congruence axiom]
BE = CE [c.p.c.t.]
and ∠3 = ∠4 [c.p.c.t.]
But ∠3 + ∠4 = 180° [a linear pair]
⇒ ∠3 = ∠4 = 90°
Hence, AD bisects BC at right angles.

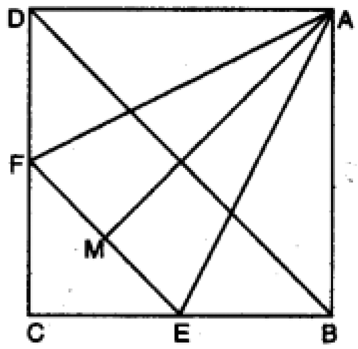
Long Answer Type

Question 1. In the given figure, AP and DP are bisectors of two adjacent angles A and D of quadrilateral ABCD. Prove that 2 ∠APD = ∠B + 2C.

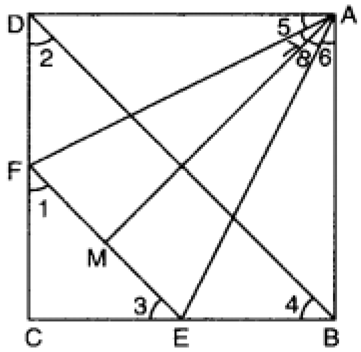


Solution:
Here, AP and DP are angle bisectors of ∠A and ∠D
∴ ∠DAP = 12∠DAB and ∠ADP = 12∠ADC ……(i)
In ΔAPD, ∠APD + ∠DAP + ∠ADP = 180°
⇒ ∠APD + 12 ∠DAB + 12∠ADC = 180°
⇒ ∠APD = 180° – 12(∠DAB + ∠ADC)
⇒ 2∠APD = 360° – (∠DAB + ∠ADC) ……(ii)
Also, ∠A + ∠B + C + ∠D = 360°
∠B + 2C = 360° – (∠A + ∠D)
∠B + C = 360° – (∠DAB + ∠ADC) ……(iii)
From (ii) and (iii), we obtain
2∠APD = ∠B + ∠C

Question 2. In figure, ABCD is a square and EF is parallel to diagonal BD and EM = FM. Prove that (i) DF = BE (i) AM bisects ∠BAD.

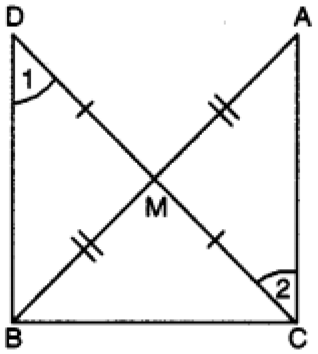


Solution:
(i) EF || BD = ∠1 = ∠2 and ∠3 = ∠4 [corresponding ∠s]
Also, ∠2 = ∠4
⇒ ∠1 = ∠3
⇒ CE = CF (sides opp. to equals ∠s of a Δ]
∴ DF = BE [∵ BC – CE = CD – CF)



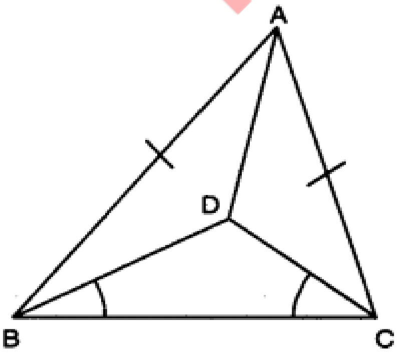
(ii) In ΔADF and ΔABE, we have
AD = AB [sides of a square]
DF = BE [proved above]
∠D = ∠B = 90°
⇒ ΔADF ≅ ΔABE [by SAS congruence axiom]
⇒ AF = AE and ∠5 = ∠6 … (i) [c.p.c.t.]
In ΔAMF and ΔAME
AF = AE [proved above]
AM = AM [common]
FM = EM (given)
∴ ΔAMF ≅ ΔAME [by SSS congruence axiom]
∴ ∠7 = ∠8 …(ii) [c.p.c.t.]
Adding (i) and (ii), we have
∠5 + ∠7 = ∠6 + ∠8
∠DAM = ∠BAM
∴ AM bisects ∠BAD.

Question 3. In right triangle ABC, right-angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see fig.). Show that : (i) ΔAMC ≅ ΔBMD (ii) ∠DBC = 90° (ii) ΔDBC ≅ ΔACB (iv) CM = 12AB



Solution:
Given : ΔACB in which 4C = 90° and M is the mid-point of AB.
To Prove :
(i) ΔAMC ≅ ΔBMD
(ii) ∠DBC = 90°
(iii) ΔDBC ≅ ΔACB
(iv) CM = 12AB
Proof : Consider ΔAMC and ΔBMD,
we have AM = BM [given]
CM = DM [by construction]
∠AMC = ∠BMD [vertically opposite angles]
∴ ΔAMC ≅ ΔBMD [by SAS congruence axiom]
⇒ AC = DB …(i) [by c.p.c.t.]
and ∠1 = ∠2 [by c.p.c.t.]
But ∠1 and ∠2 are alternate angles.
⇒ BD || CA
Now, BD || CA and BC is transversal.
∴ ∠ACB + ∠CBD = 180°
⇒ 90° + CBD = 180°
⇒ ∠CBD = 90°
In ΔDBC and ΔACB,
we have CB = BC [common]
DB = AC [using (i)]
∠CBD = ∠BCA
∴ ΔDBC ≅ ΔACB
⇒ DC = AB
⇒ 12AB = 12DC
⇒ 12AB = CM or CM = 12AB (∵ CM = 12DC)

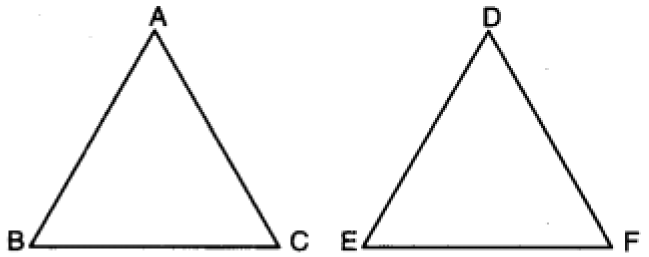
Question 4. In figure, ABC is an isosceles triangle with AB = AC. D is a point in the interior of ΔABC such that ∠BCD = ∠CBD. Prove that AD bisects ∠BAC of ΔABC.



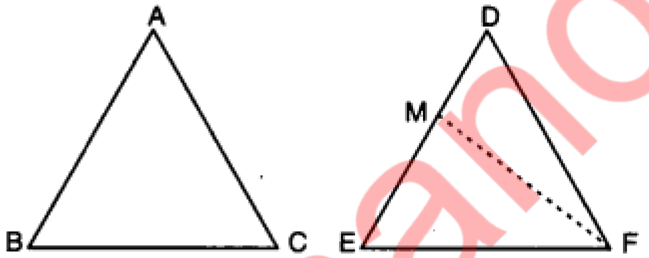
Solution:
In ΔBDC, we have ∠DBC = ∠DCB (given).
⇒ CD = BD (sides opp. to equal ∠s of ΔDBC)
Now, in ΔABD and ΔACD,

we have AB = AC [given]
BD = CD [proved above]
AD = AD [common]
∴ By using SSS congruence axiom, we obtain
ΔABD ≅ ΔACD
⇒ ∠BAD = ∠CAD [c.p.ç.t.]
Hence, AD bisects ∠BAC of ΔABC.

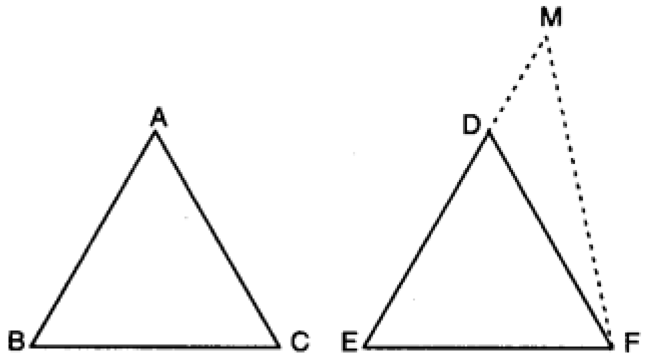
Question 5. Prove that two triangles are congruent if any two angles and the included side of one triangle is equal to any two angles and the included side of the other triangle.
Solution:



Given : Two As ABC and DEF in which
∠B = ∠E,
∠C = ∠F and BC = EF
To Prove : ΔABC = ΔDEF
Proof : We have three possibilities
Case I. If AB = DE,
we have AB = DE,
∠B = ∠E and BC = EF.
So, by SAS congruence axiom, we have ΔABC ≅ ΔDEF

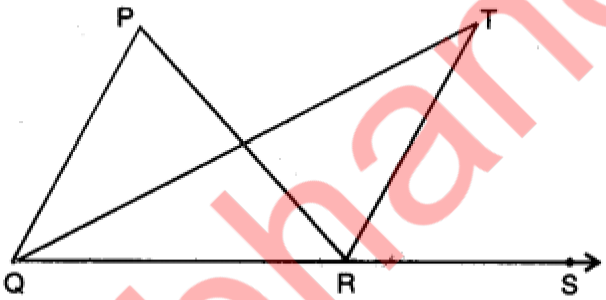


Case II. If AB < ED, then take a point Mon ED
such that EM = AB.
Join MF.
Now, in ΔABC and ΔMEF,
we have
AB = ME, ∠B = ∠E and BC = EF.
So, by SAS congruence axiom,
we have ΔABC ≅ ΔMEF
⇒ ∠ACB = ∠MFE
But ∠ACB = ∠DFE
∴ ∠MFE = ∠DFE



Which is possible only when FM coincides with B FD i.e., M coincides with D.
Thus, AB = DE
∴ In ΔABC and ΔDEF, we have
AB = DE,
∠B = ∠E and BC = EF
So, by SAS congruence axiom, we have
ΔABC ≅ ΔDEF
Case III. When AB > ED
Take a point M on ED produced
such that EM = AB.
Join MF
Proceeding as in Case II, we can prove that
ΔABC = ΔDEF
Hence, in all cases, we have
ΔABC = ΔDEF.

Question 6. In the given figure, side QR is produced to the point S. If the bisectors of ∠PQR and ∠PRS meet at T,
prove that ∠QTR = 12 ∠QPR.



Solution:
Here, QT is angle bisector of ∠PQR

$$\Rightarrow \quad \angle PQT = \angle TQR = \frac{1}{2}\angle PQR \quad ...(i)$$

Similarly, RT is angle bisector of ∠PRS

$$\Rightarrow \quad \angle PRT = \angle TRS = \frac{1}{2}\angle PRS \quad ...(ii)$$

In ΔQTR ext.∠TRS = ∠QTR + ∠TQR
or ∠QTR = ∠TRS – ∠TQR

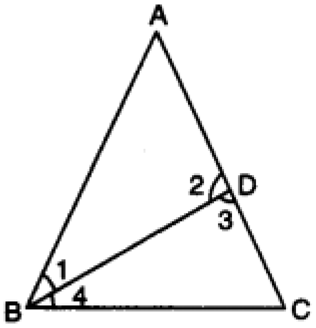
$$= \frac{1}{2}\angle PRS - \frac{1}{2}\angle PQR \quad \text{[using (i) and (ii)]}$$

$$= \frac{1}{2}(\angle PRS - \angle PQR)$$

$$= \frac{1}{2}\angle QPR \quad \text{[using ext. angle property in } \Delta PQR]$$

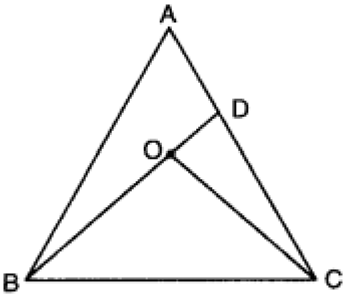
Extra Questions HOTS

Question 1. Show that the difference of any two sides of a triangle is less than the third side.
Solution:



Consider a triangle ABC
To Prove :
(i) AC – AB < BC
(ii) BC – AC < AB
(iii) BC – AB < AC
Construction : Take a point D on AC
such that AD = AB.
Join BD.
Proof : In ΔABD, we have ∠3 > ∠1 …(i)
[∵ exterior ∠ is greater than each of interior opposite angle in a Δ]
Similarly, in ΔBCD, we have
∠2 > ∠4 …..(ii) [∵ ext. ∠ is greater then interior opp. angle in a Δ]
In ΔABD, we have
AD = AB [by construction]
∠1 = ∠2 …(iii) [angles opp. to equal sides are equal in a triangle]
From (i), (ii) and (iii), we have
⇒ ∠3 > ∠4 =
⇒ BC > CD
⇒ CD < BC
AC – AD < BC
AC – AB < BC [∵ AD = AB]
Hence, AC – AB < BC
Similarly, we can prove
BC – AC < AB
and BC – AB < AC

Question 2. In the figure, O is the interior point of ΔABC. BO meets AC at D. Show that OB + OC < AB + AC.

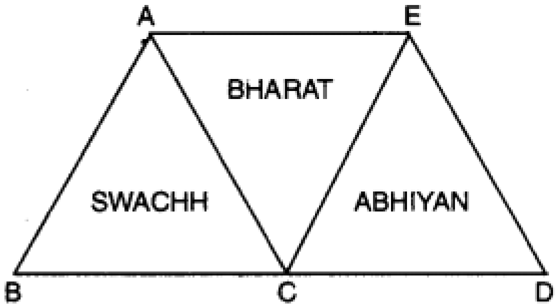


Solution: In ΔABD, AB + AD > BD …(i)
∵ The sum of any two sides of a triangle is greater than the third side. Also, we have
BD = BO + OD
AB + AD > BO + OD ….(ii)
Similarly, in ΔCOD, we have

OD + DC > OC … (iii)
On adding (ii) and (iii), we have
AB + AD + OD + DC > BO + OD + OC
⇒ AB + AD + DC > BO + OC
⇒ AB + AC > OB + OC
or OB + OC < AB + AC
Hence, proved.

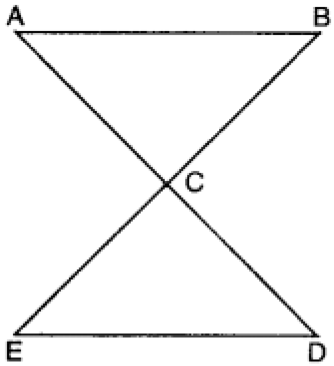
Value Based Questions (VBQs)

Question 1. A campaign is started by volunteers of mathematical club to boost school and its surrounding under Swachh Bharat Abhiyan. They made their own logo for this campaign. What values are acquired by mathematical club ?
If it is given that ΔABC ≅ ΔECD, BC = AE.
Prove that ΔABC ≅ ΔCEA.



Solution:
Here, it is given that
ΔABC ≅ ΔECD
AB = CE [c.p.c.t.]
BC = CD [c.p.c.t.]
AC = ED [c.p.c.t.]
Now, in ΔABC and ΔCEA
BC = AE [given]
AB = EC [proved above]
AC = AC [common]
∴ By using SSS congruence axiom, we have
ΔABC ≅ ΔCEA
Value : Cleanliness and social concerning.

Question 2. Rajiv, a good student and actively involved in applying knowledge A of mathematics in daily life. He asked his classmate Rahul to make triangle as shown by choosing one of the vertex as common. Rahul tried but not correctly. After sometime Rajiv hinted Rahul about congruency of triangle. Now, Rahul fixed vertex C as common vertex and locate point D, E such that AC = CD and BC = CE. Was the triangle made by Rahul is congruent ? Write the condition satisfying congruence. What value is depicted by Rajiv's action?



Solution: In ΔABC and ΔDEC, we have
AC = DC [by construction]
BC = EC [by construction]
∠ACB = ∠ECD [vert. opp. ∠s]
By using SAS congruence axiom, we have
ΔABC ≅ ΔDEC
Value : Cooperative learning, use of concept and friendly nature.